1. (25 points) Consider a theory of two scalar fields, $\phi_1$ and $\phi_2$, with potential

$$V(\phi_1, \phi_2) = (\phi_1^2 - \sigma^2)^2 + (\phi_2^2 - (\phi_1 - \sigma)^2)^2.$$ 

Write down the equations of motion for this theory. What are the possible zero-energy (vacuum) values of the fields? What kinds of domain walls are there in this theory?

2. (25 points) Imagine that cosmic strings are created by a phase transition at a temperature $T \sim \sigma$, with a tension $\mu \sim \sigma^2$ (ignoring dimensionless parameters). But imagine that the interactions of the strings are such that they do not easily intercommute and chop up into loops, but instead get tangled up with each other, so that the overall string energy density redshifts as $\rho_S \propto a^{-2}$. What would the energy scale $\sigma$ have to be for the strings to comprise the dark energy today (i.e., $\Omega_S = 0.7$)? Approximately how likely is it that one such string is passing through the Solar System right now?

3. (50 points) Consider inflation with a potential $V(\phi) = \frac{1}{2} m^2 \phi^2$. Imagine that inflation begins right at the Planck scale, with $\rho \sim M^4_{\text{pl}}$.

   (a) Show that the field obeys $\phi = \phi_0 - \beta t$, and solve for $\beta$.

   (b) Show that the inflationary solution is stable to homogeneous perturbations. That is, let $\phi(t) = \bar{\phi}(t) + \chi(t)$, where $\chi$ is a small perturbation. Derive an equation for $\chi$, and show that it doesn’t grow with time.