Physics 371: Problem Set 7

Sean Carroll, Spring 2006

Due Thursday 1 June, 2:00 p.m.

1. (25 points) Consider a theory of two scalar fields, ϕ_1 and ϕ_2, with potential

$$V(\phi_1, \phi_2) = (\phi_1^2 - \sigma^2)^2 + (\phi_2^2 - (\phi_1 - \sigma)^2)^2 \ .$$

   Write down the equations of motion for this theory. What are the possible zero-energy (vacuum) values of the fields? What kinds of domain walls are there in this theory?

2. (25 points) Imagine that cosmic strings are created by a phase transition at a temperature $T \sim \sigma$, with a tension $\mu \sim \sigma^2$ (ignoring dimensionless parameters). But imagine that the interactions of the strings are such that they do *not* easily intercommute and chop up into loops, but instead get tangled up with each other, so that the overall string energy density redshifts as $\rho_{\rm S} \propto a^{-2}$. What would the energy scale σ have to be for the strings to comprise the dark energy today (i.e., $\Omega_{\rm S} = 0.7$)? Approximately how likely is it that one such string is passing through the Solar System right now?

3. (50 points) Consider inflation with a potential $V(\phi) = \frac{1}{2}m^2\phi^2$. Imagine that inflation begins right at the Planck scale, with $\rho \sim M_{pl}^4$.

   (a) Show that the field obeys $\phi = \phi_0 - \beta t$, and solve for β.

   (b) Show that the inflationary solution is stable to homogeneous perturbations. That is, let $\phi(t) = \bar{\phi}(t) + \chi(t)$, where χ is a small perturbation. Derive an equation for χ, and show that it doesn't grow with time.