Physics 264: Problem Set 7

Sean Carroll, Fall 2005

Due Thursday 17 November, 12:00 noon

1. (Hartle 9-7; 30 points) Two particles fall radially in from infinity in the Schwarzschild geometry. One starts with energy per unit mass $e = 1$, the other with $e = 2$. A stationary observer at $r = 6GM$ measures the speed of each when they pass by. How much faster is the second particle moving at that point?

2. (Hartle 9-11; 40 points) A small perturbation of an unstable circular orbit will grow exponentially in time. Show that a displacement δr from the unstable maximum of the Schwarzschild effective potential will grow initially as $\delta r \propto e^{\tau/\tau_*}$, where τ is the proper time along the particle's trajectory and τ_* is a constant. Evaluate τ_*. Explain its behavior as the radius of the orbit approaches $6GM$.

3. (Hartle 9-14; 30 points) In Newtonian mechanics, one of Kepler's laws says that equal areas are swept out in equal times as a particle moves in an elliptical orbit in a $1/r$ potential. Consider the area outside a radius $R > 2GM$ that is swept out by an orbit in the Schwarzschild metric that stays outside this radius. Does Kepler's law hold true, using either Schwarzschild coordinate time or proper time?