Physics 364: Problem Set 7

Sean Carroll, Winter 2005

Due Wednesday 2 March, 12:00 noon.

1. (30 points) Recall that "static" means there is a timelike Killing vector which is orthogonal to spacelike hypersurfaces.

   (a) Generally speaking, if a vector field v^μ is orthogonal to a set of hypersurfaces defined by $f =$ constant, then we can write the vector as $v_\mu = h\nabla_\mu f$ (here both f and h are functions). Show that this implies

   $$v_{[\sigma}\nabla_\mu v_{\nu]} = 0 \ .$$

   (b) Imagine we have a perfect fluid with zero pressure (dust), which generates a solution to Einstein's equations. Show that the metric can be static only if the fluid four-velocity is parallel to the timelike (and hypersurface-orthogonal) Killing vector.

   (c) Use Raychaudhuri's equation to prove that there is no static solution to Einstein's equations in the presence of a perfect fluid if the pressure is zero and the energy density is greater than zero.

2. (20 points) Consider Einstein's equations in vacuum, but with a cosmological constant, $G_{\mu\nu} = \Lambda g_{\mu\nu}$.

   (a) Solve for the most general static, spherically symmetric metric, in coordinates (t, r) that reduce to ordinary Schwarzschild coordinates when $\Lambda = 0$.

   (b) Write down the equation of motion for radial geodesics in terms of an effective potential. Sketch the effective potential for massive particles, and briefly describe its features in words.