Physics 371: Problem Set 5

Sean Carroll, Spring 2006

Due Thursday 11 May, 1:30 p.m.

1. (50 points) Consider a massive particle X (equal in number to $\bar{X}$), which has a non-relativistic annihilation cross-section

$$\sigma_{X\bar{X}} = \left(\frac{m_N}{m_X}\right)^2 \sigma_{N\bar{N}} ,$$

where $\sigma_{N\bar{N}} = (90 \text{ GeV}^{-2})/v$ is a typical nucleon annihilation cross-section.

   (a) For what mass m_X will $\Omega_{X0} = 1$?

   (b) What is the number density today?

   (c) Assuming that the local density of such particles is equal to the average cosmological density, and that a typical particle velocity is 300 km/sec, what would the cross-section for σ_{XN} interactions of X with nucleons need to be to see one interaction per day in a one-ton terrestrial detector? Assume that the cross-section of X with nuclei of atomic number A is just $A\sigma_{XN}$, that $\sigma_{\bar{X}N} = \sigma_{XN}$, and that we can ignore backgrounds.

2. (50 points) Assume that the contribution of Standard-Model particles to the effective number of relativistic degrees of freedom during BBN is $\bar{g}_* \approx 10$, that the conversion of free neutrons and protons into ^{4}He takes place at $T_{\rm nuc} = 0.1$ MeV, and that the weak interactions keeping protons and neutrons in equilibrium have a rate

$$\Gamma = 2.0 \left(\frac{T}{\text{MeV}}\right)^5 \ \text{sec}^{-1} .$$

   (a) Derive how the time of nucleosynthesis $t_{\rm nuc}$, the neutron-proton freeze-out temperature T_f, and the helium abundance X_4 depend on the speed-up factor $\zeta = H/\bar{H}$.

   (b) What is the percentage change in the final helium abundance if there is a new massless fermion species that decouples at 2 MeV? What if it decouples at 400 GeV?

   (c) What is the percentage change in the final helium abundance if Newton's constant G is twice as big during nucleosynthesis as it is today?