Physics 264: Problem Set 5

Sean Carroll, Fall 2005

Due Thursday 3 November, 12:00 noon

1. (Hartle 7-10; 25 points) An observer moves on a curve $X = 2T$ for $T > 1$ in a two-dimensional geometry with metric

$$ds^2 = -X^2 dT^2 + dX^2 \tag{1}$$

   (a) What are the components of the four-velocity of the observer? Is this curve timelike?

   (b) Find the components of an orthonormal basis $(\mathbf{e}_{(\hat{0})}, \mathbf{e}_{(\hat{1})})$ for this observer, such that the four-velocity is parallel to $\mathbf{e}_{(\hat{0})}$.

2. (25 points) Show that an initially-timelike geodesic remains timelike. Do this by explicitly evaluating the derivative of the norm of the four-velocity,

$$\frac{d}{d\tau}\left(g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\right) \tag{2}$$

   and showing that it vanishes if the geodesic equation is satisfied.

3. (25 points) Consider a sphere with coordinates (θ, ϕ) and metric

$$ds^2 = d\theta^2 + \sin^2\theta\, d\phi^2 \ . \tag{3}$$

   (a) Calculate the Christoffel symbols.

   (b) Show that lines of constant longitude ($\phi =$ constant) are geodesics, and that the only line of constant latitude ($\theta =$ constant) that is a geodesic is the equator ($\theta = \pi/2$).

4. (25 pts) Consider a metric

$$ds^2 = -\left(1 - \frac{R}{r}\right)^2 \mathrm{d}t^2 + \left(1 - \frac{R}{r}\right)^{-2} \mathrm{d}r^2 + r^2\left[\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right] \ . \tag{4}$$

   where r is a radial coordinate and R is some fixed constant.

   (a) Sketch the light cones in the t-r plane (for $r > R$).

   (b) Calculate the Christoffel symbols for this metric.