Physics 371: Problem Set 4

Sean Carroll, Spring 2006

Due Thursday 4 May, 1:30 p.m.

1. (30 points) Sometimes we tell people that we can't see more than 14 billion light years away, because the universe is only 14 billion years old. This is a lie, because the universe is expanding and distances are more subtle. Derive a general formula in terms of the various density parameters Ω_i for the distance *today* (that is, the instantaneous physical distance along a spatial hypersurface) between us and an object we observe at redshift z. Then give numerical answers in light-years for objects at $z = 6$ (a distant quasar) and $z = 1200$ (the microwave background), assuming that the universe is completely matter-dominated and $H_0 = 70$ km/sec/Mpc.

2. (30 points) By what factor did the scale factor a increase between the time when the temperature of the universe was 150 MeV and when it was 10 MeV?

3. (40 points) Suppose that there is a new spin-0 boson Φ, nearly massless, which couples to neutrinos and mediates new interactions between them, $\nu\nu \leftrightarrow \Phi \leftrightarrow \bar{\nu}\bar{\nu}$. This interaction gives very tiny masses ($<< 1$ keV) to the neutrinos, so that $g_\nu = g_{\bar{\nu}} = 2$. Imagine that this interaction freezes out at $T_f = 1$ keV. What is the value of $\langle \sigma v \rangle$ at T_f (for the $\nu\nu \leftrightarrow \bar{\nu}\bar{\nu}$ process)? Note that this interaction violates lepton number, but we're imagining that this is okay for now. (In principle it is, if we have so-called "Majorana masses" for the neutrinos.) Also, what is T_ν / T_γ in this model today?