Physics 371: Problem Set 4
Sean Carroll, Spring 2006
Due Thursday 4 May, 1:30 p.m.

1. (30 points) Sometimes we tell people that we can’t see more than 14 billion light years away, because the universe is only 14 billion years old. This is a lie, because the universe is expanding and distances are more subtle. Derive a general formula in terms of the various density parameters $\Omega_i$ for the distance today (that is, the instantaneous physical distance along a spatial hypersurface) between us and an object we observe at redshift $z$. Then give numerical answers in light-years for objects at $z = 6$ (a distant quasar) and $z = 1200$ (the microwave background), assuming that the universe is completely matter-dominated and $H_0 = 70 \text{ km/sec/Mpc}$.

2. (30 points) By what factor did the scale factor $a$ increase between the time when the temperature of the universe was 150 MeV and when it was 10 MeV?

3. (40 points) Suppose that there is a new spin-0 boson $\Phi$, nearly massless, which couples to neutrinos and mediates new interactions between them, $\nu \nu \leftrightarrow \Phi \leftrightarrow \bar{\nu} \bar{\nu}$. This interaction gives very tiny masses ($<< 1 \text{ keV}$) to the neutrinos, so that $g_\nu = g_{\bar{\nu}} = 2$. Imagine that this interaction freezes out at $T_f = 1 \text{ keV}$. What is the value of $\langle \sigma v \rangle$ at $T_f$ (for the $\nu \nu \leftrightarrow \bar{\nu} \bar{\nu}$ process)? Note that this interaction violates lepton number, but we’re imagining that this is okay for now. (In principle it is, if we have so-called “Majorana masses” for the neutrinos.) Also, what is $T_\nu/T_\gamma$ in this model today?