Physics 264: Problem Set 4

Sean Carroll, Spring 2004

Due Thursday 27 October, 12:00 noon

1. (Hartle 6-9; 40 points) A GPS satellite emits signals at a constant rate as measured by an on-board clock. Calculate the fractional difference in the rate at which these are received by an identical clock on the surface of the Earth. Take both the effects of special relativity and gravitation into account, to leading order in $1/c^2$. For simplicity, assume the satellite is in a circular equatorial orbit, the ground-based clock is on the equator, and that the angle between the propagation of the signal and the velocity of the satellite is $90°$ in the instantaneous rest frame of the receiver. (Satellite orbit parameters can be found on page 124 of Hartle, copied on the next page.)

2. (Hartle 6-13; 30 points) Three observers are standing near each other on the surface of the Earth. Each holds an accurate atomic clock. At time $t = 0$ the first observer throws their clock straight up so that it returns at time T as measured by the clock of the second observer, who holds their clock in their hand for the entire time interval. The third observer carries their clock up to the maximum height reached by the thrown clock, and back down, moving with constant speed on each leg of the trip and returning in time T.

   Calculate the total elapsed time measured on each clock assuming that the maximum height is much smaller than the radius of the Earth. Include gravitational effects but calculate to order $1/c^2$ only using non-relativistic trajectories. Which clock registers the longest proper time? Why is this?

3. (30 points) Consider Euclidean three-dimensional space with Cartesian coordinates (x, y, z). Introduce paraboloidal coordinates (u, v, ϕ) via

$$x = uv\cos\phi \qquad y = uv\sin\phi \qquad z = \frac{1}{2}(u^2 - v^2)\ . \tag{1}$$

   (a) Sketch what surfaces of constant u, v, and ϕ look like.

   (b) Calculate the metric $g_{\mu'\nu'}$ in the new coordinates.

   (c) Calculate the inverse metric $g^{\mu'\nu'}$ in the new coordinates.