Physics 371: Problem Set 3

Sean Carroll, Spring 2006

Due Thursday 27 April, 1:30 p.m.

1. Define $a_{\rm eq}$ as the scale factor and $t_{\rm eq}$ as the time when the density of radiation $\rho_{\rm R}$ (including massless neutrinos) equals the density of matter $\rho_{\rm M}$. Assume a flat universe, $k = 0$.

   (a) Show that
   $$\frac{t}{t_{\rm eq}} = \frac{1}{2-\sqrt{2}}\left[\left(\frac{a}{a_{\rm eq}} - 2\right)\left(\frac{a}{a_{\rm eq}} + 1\right)^{1/2} + 2\right] . \tag{1}$$

   (b) Show that
   $$H_{\rm eq} = \frac{4}{3}(\sqrt{2}-1)t_{\rm eq}^{-1} . \tag{2}$$

   (c) Calculate $1 + z_{\rm eq} = a_0/a_{\rm eq}$ from
   $$\rho_{\rm R0} = \frac{\pi^2}{30} g_{*0} T_0^4 . \tag{3}$$
   Express your answer in terms of $\Omega_{\rm M0}$ and $h = H_0/(100\ \text{km/sec/Mpc})$. That is, use your knowledge of T_0 and g_{*0}.

   (d) Calculate the photon temperature $T_{\rm eq}$ in terms of h.

2. This problem is considerably simplified in comparison with reality, but it's enought to get an idea of how things work. We imagine that a massless electron neutrino with 100 GeV of energy scatters off of a proton at rest in our lab, leaving the identity of each particle unaltered. For purposes of this problem, imagine that the proton is a single featureless particle (rather than made up of quarks), and that the weak-interaction vertex comes with a factor $g_w \sim 0.6$ for both the neutrino and the proton.

   (a) What is the center-of-mass energy, $E_{\rm cm} = \sqrt{s}$?

   (b) What Feynman diagram gives the largest contribution to this process?

   (c) Estimate the cross-section using the techniques discussed in class. Express your answer in barns (1 barn $= 10^{-24}$ cm^2).

   (d) Calculate the equivalent cross-section if the particle at rest in our lab had been an electron rather than a proton. Which is larger?

(e) In either case (electron or proton), would the answer have been different if it had been a muon neutrino rather than an electron neutrino?

3. Section 3.4 of Kolb and Turner gives a sketch of how to derive the entropy per comoving volume, $S = a^3(\rho + p)/T$. Derive the generalization of this formula to the case of a nonzero chemical potential μ, starting from

$$TdS = d(\rho V) + pdV - \mu d(nV) \; . \tag{4}$$

Be explicit.