Physics 264: Problem Set 3

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Due Thursday 20 October, 12:00 noon

1. (30 points) Quasar 3C 272 emits relativistic blobs of plasma from near a massive black hole at its center. The blobs travel with speed $v = dx/dt$ at an angle θ with respect to the line-of-sight of the observer. Projected onto the sky, the blobs appear to travel perpendicular to the line of sight with angular speed $v_{\rm app}/r$, where r is the distance to the quasar (treating space as Euclidean, *i.e.* ignoring the expansion of the universe) and $v_{\rm app}$ is the "apparent speed."

   (a) Show that
   $$v_{\rm app} = \frac{v \sin\theta}{1 - v\cos\theta} \ . \tag{1}$$

   (b) For a given value of v, what value of θ maximizes the apparent speed? Can $v_{\rm app}$ exceed 1 (the speed of light)?

   (c) For 3C 273, $v_{\rm app} \approx 10$. What is the largest value of θ in radians?

2. (20 points) A cosmic-ray proton (mass 940 MeV) travels through space at high velocity. If the center-of-mass energy is high enough, it can collide with a cosmic microwave background (CMB) photon (the temperature of the CMB is 2.74K in its overall rest frame) and convert into a proton plus a neutral pion (mass 140 MeV). The pion will then decay into unobservable particles, while the proton will have a lower energy than before the collision. What is the cosmic-ray energy above which we expect this process to occur, and therefore provide a cutoff in the cosmic-ray spectrum? (This is known as the Griesen-Zatsepin-Kuzmin, or GZK, cutoff. In fact there are observational indications that it is violated, which might be a sign of new physics — even, some have suggested, a violation of special relativity.)

3. (50 points) A common occurence in particle physics is the scattering of two particles $A+B$ into two new particles $C+D$. For such events it is convenient to define *Mandelstam variables*:
$$\begin{aligned} s &= -\eta_{\mu\nu}(p^\mu_A + p^\mu_B)(p^\nu_A + p^\nu_B) \\ t &= -\eta_{\mu\nu}(p^\mu_A - p^\mu_C)(p^\nu_A - p^\nu_C) \\ u &= -\eta_{\mu\nu}(p^\mu_A - p^\mu_D)(p^\nu_A - p^\nu_D) \ , \end{aligned}$$

where p_i^μ are the 4-momenta. The beauty of these variables is that they are all Lorentz-invariant.

(a) Show that $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$.

(b) Express the energy of A in the center-of-mass (CM) frame (in which the spatial components of the total momentum vanish), in terms of the masses and the Mandelstam variables.

(c) Express the energy of A in the "lab" frame, in which B is at rest.

(d) Express the total energy in the CM frame.

(e) For scattering of identical particles, $A + A \to A + A$, show that in the CM frame we have

$$\begin{aligned} s &= 4(\vec{p}^2 + m_A^2) \\ t &= -2\vec{p}^2(1 - \cos\theta) \\ u &= -2\vec{p}^2(1 + \cos\theta) \ , \end{aligned}$$

where $\vec{p}$ is the 3-momentum of one of the incident particles, and θ is the scattering angle.