Physics 371: Problem Set 2

Sean Carroll, Spring 2006

Due Thursday 20 April, 1:30 p.m.

1. (30 points) This problem asks you to do some numerical evolution of the Friedmann equation and make some plots. You can use Mathematica or whatever other software you like.

   (a) Plot the scale factor vs. time for FRW universes with the following choices for the parameters ($\Omega_{\rm M}$, $\Omega_{\rm R}$, Ω_Λ): (1, 0, 0); (0, 1, 0); (0.1, 0, 0); (5, 0, 0); and (0.3, 0, 0.7). You can solve the Friedmann equation numerically, and plot your curves on the same graph, with the same value of the Hubble constant today.

   (b) Consider the best-fit universe, with density parameters $\Omega_{\rm R0} = 10^{-4}$, $\Omega_{\rm M0} = 0.3$, $\Omega_{\rm M0} = 0.7$. Make a plot of the three Ω_i's as a function of the scale factor a, on a log scale, from $a = 10^{-35}$ to $a = 10^{35}$. Indicate the Planck time, nucleosynthesis, and today.

2. (40 points) Suppose there exists a population of luminous sources distributed evenly throughout space, all of them with an identical intrinsic luminosity L, emitting isotropically (equally in all directions). Assume also that you are able to observe photons at all wavelengths (i.e. the "bolometric luminosity"), so we don't have to worry about losing luminosity due to limitations of our observing apparatus. Let the comoving number density of these sources be called $n(t)$; the physical number density will of course scale as a^{-3} in addition to any intrinsic evolution of $n(t)$. If we observe the entire sky, how many sources $N(F_*)$ should we observe with a measured flux greater than F_*? Evaluate it fully for the simple case of no evolution ($n(t) =$ constant) in a flat matter-dominated universe. Show that as F_* becomes very large, you recover the result you would get in an unexpanding Minkowski space.

3. (30 points) In a flat spacetime, objects of a fixed physical size subtend smaller and smaller angles as they are further and further away; in an expanding universe this is not necessarily so. Consider the angular size $\theta(z)$ of an object of physical size L at redshift z. In a matter-dominated flat universe, at what redshift is $\theta(z)/L$ a minimum? If all galaxies are at least 10 kpc across (and always have been), what is the minimum angular size of a galaxy in such a universe? (Express your result both in terms of H_0, and plugging in $H_0 = 70$ km/s/Mpc.)