Physics 264: Problem Set 2

Sean Carroll, Fall 2005

Due Thursday 13 October, 12:00 noon

1. (Hartle 4-2; 25 points) A rocket ship of proper length (*i.e.*, length in its rest frame) L leaves the Earth vertically at speed $(4/5)c$. A light signal is sent vertically, after which it arrives at the rocket's tail at $t = 0$ according to both the rocket- and Earth-based clocks. When does the signal reach the nose of the rocket according to (a) the rocket clocks; (b) the Earth clocks?

2. (Hartle 4-13; 25 points) In an inertial laboratory frame, two events occur simultaneously at a distance of 3 meters apart. In a frame moving with respect to the laboratory frame, one event occurs later than the other by 10^{-8} s. By what spatial distance are the two events separated in the moving frame? Solve this problem in two ways: first by finding the Lorentz boost that connects the two frames, and second by making use of the invariance of the spacetime interval between the two events.

3. (Hartle 5-2; 25 points) The scalar product between two three-vectors can be written
$$\vec{a} \cdot \vec{b} = ab\cos\theta \ , \tag{1}$$
where θ is the angle between the vectors and a and b are their lengths ($a = \sqrt{\vec{a} \cdot \vec{a}}$, etc). Show that an analogous formula holds for two timelike four-vectors $\mathbf{a}$ and $\mathbf{b}$:
$$\mathbf{a} \cdot \mathbf{b} = -ab\cosh\phi \ , \tag{2}$$
where ϕ is the boost parameter between the vectors and a and b are their lengths ($a = \sqrt{-\mathbf{a} \cdot \mathbf{a}}$, etc).

4. (Hartle 5-6; 25 points) Consider a particle moving along the x-axis whose velocity as a function of time is
$$v = \frac{dx}{dt} = \frac{gt}{\sqrt{1 + g^2t^2}} \tag{3}$$
for some constant g.

   (a) Does the particle's speed ever exceed that of light?

   (b) Calculate the components of the particle's four-velocity.

   (c) Express x and t as functions of the proper time along the trajectory.

   (d) What are the components of the four-force and the three-force acting on the particle?