1. (Hartle 4-2; 25 points) A rocket ship of proper length (i.e., length in its rest frame) $L$ leaves the Earth vertically at speed $(4/5)c$. A light signal is sent vertically, after which it arrives at the rocket’s tail at $t = 0$ according to both the rocket- and Earth-based clocks. When does the signal reach the nose of the rocket according to (a) the rocket clocks; (b) the Earth clocks?

2. (Hartle 4-13; 25 points) In an inertial laboratory frame, two events occur simultaneously at a distance of 3 meters apart. In a frame moving with respect to the laboratory frame, one event occurs later than the other by $10^{-8}$ s. By what spatial distance are the two events separated in the moving frame? Solve this problem in two ways: first by finding the Lorentz boost that connects the two frames, and second by making use of the invariance of the spacetime interval between the two events.

3. (Hartle 5-2; 25 points) The scalar product between two three-vectors can be written

$$\vec{a} \cdot \vec{b} = ab \cos \theta ,$$  \hspace{1cm} (1)

where $\theta$ is the angle between the vectors and $a$ and $b$ are their lengths ($a = \sqrt{\vec{a} \cdot \vec{a}}$, etc). Show that an analogous formula holds for two timelike four-vectors $a$ and $b$:

$$a \cdot b = -ab \cosh \phi ,$$  \hspace{1cm} (2)

where $\phi$ is the boost parameter between the vectors and $a$ and $b$ are their lengths ($a = \sqrt{-\vec{a} \cdot \vec{a}}$, etc).

4. (Hartle 5-6; 25 points) Consider a particle moving along the $x$-axis whose velocity as a function of time is

$$v = \frac{dx}{dt} = \frac{gt}{\sqrt{1 + g^2 t^2}}$$  \hspace{1cm} (3)

for some constant $g$.

(a) Does the particle’s speed ever exceed that of light?

(b) Calculate the components of the particle’s four-velocity.

(c) Express $x$ and $t$ as functions of the proper time along the trajectory.

(d) What are the components of the four-force and the three-force acting on the particle?