1. (Hartle 2-5; 25 points) Calculate the area of a circle of radius $r$ (distance from center to circumference) in the two-dimensional geometry which is the surface of a sphere of radius $a$. Show that this reduces to $\pi r^2$ when $r \ll a$.

2. (Hartle 2-7; 50 points) Consider the following coordinate transformation from the familiar rectangular coordinates $(x, y)$ labeling points in the plane to a new set of coordinates $(\mu, \nu)$

$$x = \mu \nu, \quad y = \frac{1}{2} (\mu^2 - \nu^2). \quad (0.1)$$

(a) Sketch the curves of constant $\mu$ and curves of constant $\nu$ in the $(x, y)$ plane.

(b) Transform the line element $ds^2 = dx^2 + dy^2$ into $(\mu, \nu)$ coordinates.

(c) Do the curves of constant $\mu$ and constant $\nu$ intersect at right angles? (Provide a justification, not just an answer.)

(d) Find the equation of a circle of radius $r$ centered at the origin in terms of $\mu$ and $\nu$.

(e) Calculate the ratio of the circumference to the diameter of a circle using $(\mu, \nu)$ coordinates. Do you get the correct answer?

3. (Hartle 3-1; 25 points) Show that Newton’s laws of motion are not invariant under a transformation to a frame that is uniformly accelerated with respect to an inertial frame. What are the equations of motion in the accelerated frame (say, accelerated in the $x$-direction)?