Cosmological Constant

The cosmological constant, conventionally denoted by the Greek letter Λ, is a parameter describing the energy density of the vacuum (empty space), and a potentially important contributor to the dynamical history of the universe. Unlike ordinary matter, which can clump together or disperse as it evolves, the energy density in a cosmological constant is a property of spacetime itself, and under ordinary circumstances is the same everywhere. A sufficiently large cosmological constant will cause galaxies to appear to accelerate away from us, in contrast to the tendency of ordinary forms of energy to slow down the recession of distant objects. The value of Λ in our present universe is not known, and may be zero, although there is some evidence for a nonzero value; a precise determination of this number will be one of the primary goals of observational cosmology in the near future.

The Cosmological Constant and Vacuum Energy

We live in an expanding universe: distant galaxies are moving away from us, such that the more distant ones are receding faster. Cosmologists describe this expansion by defining a scale factor $R(t)$, which specifies the relative distance of galaxies as a function of time: when the value of the scale factor doubles, the distance between any two galaxies doubles. The behavior of the scale factor is governed by the curvature of space (which can be positive, negative, or zero) and the average energy density of the universe (which is thought to be positive, although we should be open to exotic possibilities).

Imagine taking a region of space and removing from it all of the matter, radiation, and other substances we could conceivably remove. The resulting state is referred to as the "vacuum" — a somewhat stricter use of the word than that applied to the space in between planets and stars, which is actually occupied by trace amounts of matter and radiation. The vacuum has the lowest energy of any state, but there is no reason in principle for that energy to be zero. In the absence of gravity there is no way of measuring energy on an absolute scale; the best we can do is to compare the relative energies of two different states. The vacuum energy is then arbitrary, unobservable. In the GENERAL THEORY OF RELATIVITY, however, any form of energy affects the gravitational field, so the vacuum energy becomes a potentially crucial ingredient. To a good approximation (see below), we believe that the vacuum is the same everywhere in the universe, so the vacuum energy density is a universal number which we call the cosmological constant. (More precisely, the conventionally defined cosmological constant Λ is proportional to the vacuum energy density ρ_Λ; they are related by $\Lambda = (8\pi G/3c^2)\rho_\Lambda$, where G is Newton's constant of gravitation and c is the speed of light. It was not until years after Einstein introduced Λ as a parameter in cosmology that it was realized that the same parameter measured the energy density of the vacuum.)

The Cosmological Constant in Cosmology

The scale factor $R(t)$, spatial curvature, and energy density of the universe are related by the Friedmann equation, which says that a positive energy density contributes positively to the curvature, while expansion contributes negatively. For simplicity, consider a flat universe — zero spatial curvature — so that the energy density and expansion are in perfect balance. (If the universe starts out with zero spatial curvature, it will remain that way throughout its evolution.) As the universe expands, the matter within it becomes increasingly rarefied, so the energy density in matter diminishes. If matter is the dominant component of the energy, the expansion rate (as measured by the HUBBLE CONSTANT) will correspondingly decrease; if on the other hand the cosmological constant dominates, the energy density will be constant, and the expansion rate will attain a constant value. In a potentially confusing but nevertheless appropriate piece of nomenclature, a universe with a constant expansion rate is said to be "accelerating". This is because, while the amount of expansion undergone in any one second by a typical cubic centimeter in such a universe is a constant, the number of centimeters between us and a distant galaxy will be increasing with time; such a galaxy will therefore be seen to have an apparent recession velocity that grows ever larger.

In a universe with both matter and vacuum energy, there is a competition between the tendency of Λ to cause acceleration and the tendency of matter to cause deceleration, with the ultimate fate of the universe depending on the precise amounts of each component. This continues to be true in the presence of spatial curvature, and with a nonzero cosmological constant it is no longer true that negatively curved ("open") universes expand indefinitely while positively curved ("closed") universes will necessarily recollapse — each of the four combinations of negative/positive curvature and eternal expansion/eventual recollapse become possible for appropriate values of the parameters. There can even be a delicate balance, in which the competition between matter and vacuum energy is a draw and the universe is static (not expanding). The search for such a solution was Einstein's original motivation for introducing the cosmological constant, as the data at the time did not indicate an expanding universe, but his solution depended on careful fine-tuning and became unnecessary once HUBBLE'S LAW was discovered. Since that time, astrophysicists have occasionally invoked a nonzero cosmological constant in order to explain puzzling observations; in the 1960's there was an apparent excess in the number of quasars at a redshift of $z \approx 1.95$, and more recently there has been disagreement between the ages of the oldest stars and that of the universe as inferred from its expansion rate. Subsequently, however, these observations have either not held up to closer scrutiny or have been explained by more conventional means.

The average energy density in the universe ρ is often expressed in terms of the density parameter Ω, defined by $\Omega = (8\pi G/3H^2c^2)\rho$, where H is the Hubble constant. The density parameter is directly related to the spatial curvature; space is negatively curved for $\Omega < 1$, flat for $\Omega = 1$, and positively curved for $\Omega > 1$. We may decompose the density parameter into a sum of contributions from different sources of energy; we therefore speak of the density parameter for matter, $\Omega_{\rm M}$, for the cosmological constant, Ω_Λ, and so on. The figure indicates the spatial curvature and future history of expanding universes as a function of $\Omega_{\rm M}$ and Ω_Λ, under the plausible (but by no means necessary) assumption that matter and vacuum energy are the only dynamically significant forms of energy in the universe today.

Note that a nonzero Ω_Λ of the same order of magnitude as $\Omega_{\rm M}$ is in a sense quite unnatural, as the relative abundance of matter and vacuum energy changes rapidly as the universe expands. Indeed, since the energy density in matter decreases as R^{-3} while that in vacuum remains constant, we have $\Omega_\Lambda/\Omega_{\rm M} \propto R^3$. To have approximate equality between these two

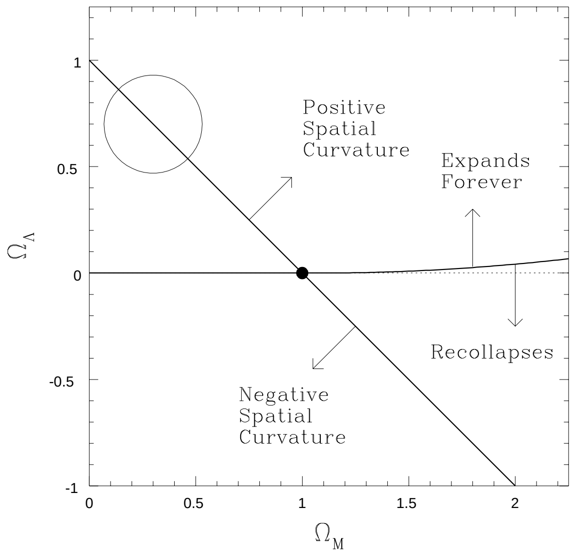

Figure 1: Geometry and evolution of universes with different amounts of matter and vacuum energy, as parameterized by the density parameters Ω_M and Ω_Λ. The diagonal line $\Omega_M + \Omega_\Lambda = 1$ represents spatially flat universes. The circle centered on $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$ represents very roughly the region favored by current observations of distant supernovae, the cosmic microwave background, and the dynamics of galaxies.

numbers at the present era would thus come as a great surprise, since the situation in the very early or very late universe would be much different.

Observational Prospects

The existence of a nonzero vacuum energy would, in principle, have an effect on gravitational physics on all scales; for example, it would alter the value of the precession of the orbit of Mercury. In practice, however, such effects accumulate over large distances, which makes cosmology by far the best venue for searching for a nonzero cosmological constant. Most of these effects depend not just on the vacuum energy but on the matter energy density as well, so a number of independent tests are necessary to pin down Ω_Λ and Ω_M separately.

There is insufficient space available to do justice to all of the ways in which we can constrain Ω_Λ, and the reader is encouraged to consult the references. A paradigmatic example is provided by the statistics of GRAVITATIONAL LENSING. A positive cosmological constant increases the volume of space in between us and a source at any fixed redshift, and therefore the probability that such a source undergoes lensing by an intervening object. Limits on the frequency with which such lensing occurs can therefore put an upper limit on Ω_Λ; current data suggest that Ω_Λ cannot be too close to 1, although upcoming surveys will provide much better data. A relatively new method for constraining various cosmological parameters, including Ω_Λ, is the analysis of temperature anisotropies in the COSMIC MICROWAVE BACKGROUND. Such anisotropies have a distinctive power on any given angular scale which can be predicted, in any specified theory of structure formation, as a function of these parameters. Observations to date have provided some preliminary evidence in favor of an approximately flat universe, $\Omega_\Lambda + \Omega_M \sim 1$, if currently favored theories based on adiabatic scale-free primordial perturbations are correct. (Most versions of the INFLATIONARY UNIVERSE scenario robustly predict that $\Omega_\Lambda + \Omega_M$ is extremely close to 1.) Coupled with dynamical tests, which consistently indicate that $0.1 \leq \Omega_M \leq 0.4$, this can be construed as evidence in favor of a nonzero cosmological constant; once again, however, these conclusions are tentative, and will soon be superseded as a new generation of telescopes and satellites provides more accurate observations of microwave background anisotropies and large-scale structure in the universe.

Perhaps the most direct way of measuring the cosmological constant is to determine the relationship between redshifts and distances of faraway galaxies, known as the HUBBLE DIAGRAM. Nearby galaxies have redshifts which are proportional to their distances (Hubble's Law), but galaxies further away are expected to deviate slightly from this strict proportionality in a way which depends on both Ω_Λ and Ω_M. Measuring the distances to cosmological objects is notoriously difficult, but important progress has recently been made by using Type Ia SUPERNOVAE as distance indicators. (Such supernovae are not precisely standard candles, but variations in their luminosity are correlated with the rate of decay of their light curves, and can be accounted for.) Supernovae are rare, but the number of distant galaxies is very large, and two independent groups have discovered dozens of high-redshift supernovae (as of 1999) by carefully observing deep into small patches of the sky. The results of these studies thus far would be consistent with zero cosmological constant only if the matter density were lower than that determined by dynamical measurements, and are consistent with a spatially flat universe only if a substantial fraction of the total energy density is due to a positive cosmological constant. It must be stressed, however, that our understanding of the physics underlying supernova explosions and the environments in which they occur is very incomplete at this stage. Nevertheless, there is an impressive consistency between this result and those of the microwave background observations and dynamical measurements of the mass density, with agreement achieved for a universe with Ω_M close to 0.3 and Ω_Λ close to 0.7. Confirming or disproving this possibility is one of the foremost ambitions of contemporary cosmologists.

Physics of the Cosmological Constant

The value of the cosmological constant is an empirical issue which will ultimately be settled by observation; meanwhile, physicists would like to develop an understanding of why the energy density of the vacuum has this value, whether it is zero or not. There are many effects which contribute to the total vacuum energy, including the potential energy of scalar fields and the energy in "vacuum fluctuations" as predicted by quantum mechanics, as well as any fundamental cosmological constant. Furthermore, many of these contributions can change with time during a phase transition; for example, we believe that the effective cosmological constant decreased by approximately 10^{47} erg cm^{-3} during the electroweak phase transition. (A change in the effective cosmological constant during a phase transition is a crucial ingredient in the inflationary universe scenario, which posits an exponential expansion in the very early universe driven by a large vacuum energy.)

From this point of view it is very surprising that the vacuum energy today, even if it is nonzero, is as small as the current limits imply ($|\rho_\Lambda| \leq 10^{-9}$ erg cm^{-3}). Either the various contributions, large in magnitude but different in sign, delicately cancel to yield an extraordinarily small final result, or our understanding of how gravitation interacts with these sources of vacuum energy is dramatically incomplete. A great deal of effort has gone into finding ways in which all of the contributions may cancel, but it is unclear what would be special about the value $\Lambda = 0$; a vanishing vacuum energy could be demanded by certain symmetry principles, but unbroken symmetries of the appropriate type are incompatible with what we know of the other forces of nature. (One suggestion is to invoke the "anthropic principle", which imagines that the constants of nature take on very different values in different regions of the universe, and intelligent observers only appear in those regions hospitable to the development of life. It is unclear, however, whether different regions of the universe really do have different fundamental constants, or what values of the cosmological constant are compatible with the existence of intelligent life.) The alternative, that our understanding of the principles underlying the calculation of the cosmological constant is insufficient (and must presumably await the construction of a complete theory of quantum gravity), is certainly plausible, although the vacuum energy manifests itself in a low-energy regime where it would have been reasonable to expect semiclassical reasoning to suffice. Understanding the smallness of the cosmological constant is a primary goal of string theory and other approaches to quantum gravity.

If the recent observational suggestions of a nonzero Λ are confirmed, we will be faced with the additional task of inventing a theory which sets the vacuum energy to a very small value without setting it precisely to zero. In this case we may distinguish between a "true" vacuum, which would be the state of lowest possible energy which simply happens to be nonzero, and a "false" vacuum, which would be a metastable state different from the actual state of lowest energy (which might well have $\Lambda = 0$). Such a state could eventually decay into the true vacuum, although its lifetime could be much larger than the current age of the universe. A final possibility is that the vacuum energy is changing with time — a dynamical cosmological "constant". This alternative, which is sometimes called "quintessence", would also be compatible with a true vacuum energy which was ultimately zero, although it appears to require a certain amount of fine-tuning to make it work. No matter which of these possibilities, if any, is true, the ramifications of an accelerating universe for fundamental physics would be truly profound.

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