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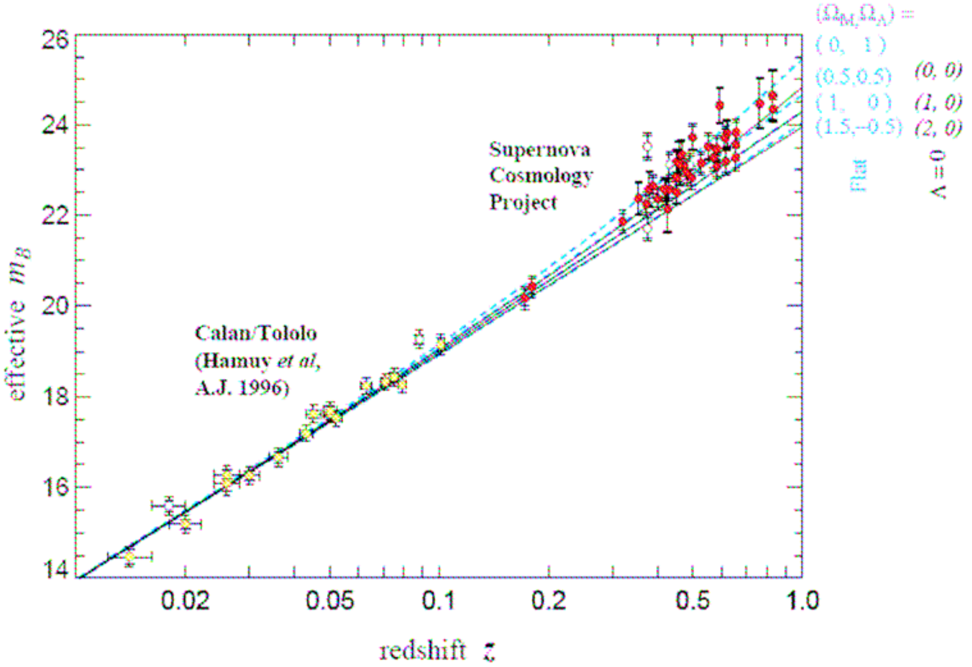
Physics 371

The Integrated Sachs-Wolfe Effect:

Evidence for Dark Energy

Any expert's list of the most important problems in physics today would certainly include the mystery that is dark energy. The discovery that a majority of the energy density in the Universe appears to be in a form previously unknown set off a flurry of research that remains unabated, and less than a decade after being found, dark energy is now a completely accepted part of the current model of cosmology. But what evidence is there? Nearly every talk that is given or class that is taught about dark energy must include the graph of the apparent magnitude of supernovae by redshift and how it shows that our Universe is clearly not matter-dominated, and is in fact accelerating in its expansion. Here is one such graph from the Perlmutter et al. 1999 paper *Measuring Ω and Λ from 42 High-Redshift Supernovae*.



Although the data has gotten better since then, with more supernovae studied, (and binning the data by redshift helps as well,) it can still be called rather underwhelming data for such a groundbreaking find. This is especially true since it all hinges on the use of Type Ia supernovae as standardizable candles, which seems to be a good assumption though is unsettling because of the vast unknowns in the physics of supernovae. There were certainly valid questions for skeptics to ask. The other evidence that is often cited is the requirement from the power spectrum of the Cosmic Microwave Background that the Universe be flat and the studies of both the CMB and local large scale structure indicating that matter makes up approximately 30% of the critical density of the Universe, leaving ~70% that must be filled by something. So with some of the data only pointing to dark energy indirectly and other data highly susceptible to systematic errors, it would be reasonable to call for a little more proof of its existence.

It turns out that there is another indication of dark energy's existence, and that is from the Integrated Sachs-Wolfe (ISW) Effect. It has been noted – at least one paper was accepted to the journal Nature, and the major papers have gotten numerous citations – but it still goes largely unmentioned in classes and talks related to dark energy, at least in this author's admittedly modest experience. It makes sense that the supernovae data are most often referred to as the main support for dark energy, since such studies of supernovae were the first major piece of evidence, are done more, and have more prospects for future study. However, from a pedagogical point of view, the ISW Effect seems more direct. In support of that argument, I will present the current status of ISW studies, going from the origin of the effect all the way through recent observations to future prospects.

An important feature of General Relativity that has been known for quite some time is that gravitational fields can alter the energies of photons and cause spectra of light to be redshifted or blueshifted, just like a relative velocity between source and observer. The Sachs-Wolfe Effect, named after Rainer Kurt Sachs and Arthur Michael Wolfe, notes how gravitational redshifting would introduce anisotropies into the Cosmic Microwave Background, since some of the photons at the Surface of Last Scattering would be coming from within gravitational potential wells caused by matter over-densities, while others would not. For the CMB, this effect acts like a temperature change. The fractional shift in temperature ΔT/T is proportional to the change in gravitational potential Φ on the photon's path. In the ordinary Sachs-Wolfe Effect, this will just be the depth of the potential well the photon was in at the time of emission on the Surface of Last Scattering. Though our telescopes are clearly in the middle of a matter over-density and hence in a potential well, any blueshift caused by falling into our potential well will affect all of the CMB equally and will not introduce any anisotropy.

The Integrated Sachs-Wolfe Effect comes from the fact that, due to structure in the Universe, there are potential wells all over the sky that the CMB photons will have fallen into and climbed out of on their way to us. One could guess that this will not matter, since any blueshift caused by falling into the over-densities will be cancelled by the redshift from climbing out. Since the shift is proportional to the change in Φ, the key factor will be to see if the depth of the well changes while the photon is crossing.

For the following analysis, I will assume that the over-density Δρ/ρ is much less than 1 and is in the form of a top-hat. I will also assume for simplicity of notation that the photon falls into the over-density at some z, and climbs out at the current epoch, z=0.

$$\Delta\phi = \frac{G(\Delta M)}{R} = G(\Delta\rho)R^2$$

It can be shown that the fractional over-density in linear perturbation theory will grow proportionally to the scale factor, so that:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta\rho_0}{\rho_0}\frac{1}{1+z}$$

Since R, being a physical size, also grows like the scale factor, we can plug in and put the equation in terms of current values.

$$\Delta\phi = G(\Delta\rho_0)R_0^2\frac{\rho}{\rho_0}\frac{1}{(1+z)^3}$$

This is the size of the drop into the potential well, while the climb out will simply be $G(\Delta\rho_o)R_o^2$. It is easy to see that in a matter-dominated Universe, where the density is proportional to $(1+z)^3$, the z factors will cancel and the blueshift on the fall in will equal the redshift on the trip out, so that there is no net effect. This would mean that large scale structure in the Universe does not affect the temperature of the CMB that we measure, and there is no ISW effect.

However, if the equation of state of the Universe changes and is no longer matter dominated, particularly if the density is constant (as would happen in a cosmological constant dominated Universe), the previous derivation would not be true. In a Λ-dominated Universe, the growth of structure is suppressed, so, to first order, ΔM stays constant since no more matter can fall into the over-density. In this scenario:

$$\Delta\phi = \frac{G(\Delta M)}{R} = \frac{G(\Delta M)}{R_0}(1+z)$$

So the depth of the potential was greater in the past when the photon fell in, and the blueshift the photon experienced at the time was greater than the redshift from coming out of the over-density later on. So while the Universe is dominated by a dark energy that accelerates the expansion, the CMB will be slightly heated after passing through any linear over-densities.

How does this play out in trying to observe the existence of dark energy? In the z<1 Universe, where dark energy would be dominating, there still are linear perturbations, but only at very large scales. At these very large scales, the CMB photons will not have traveled through many over- and under-densities, so it should be possible to match large structures nearby in the Universe with hot spots on the CMB that have passed through them. It was also predicted that the ISW effect from passing through large perturbations would add a small bump to the CMB power spectrum at large angles, but this has not been seen by WMAP.

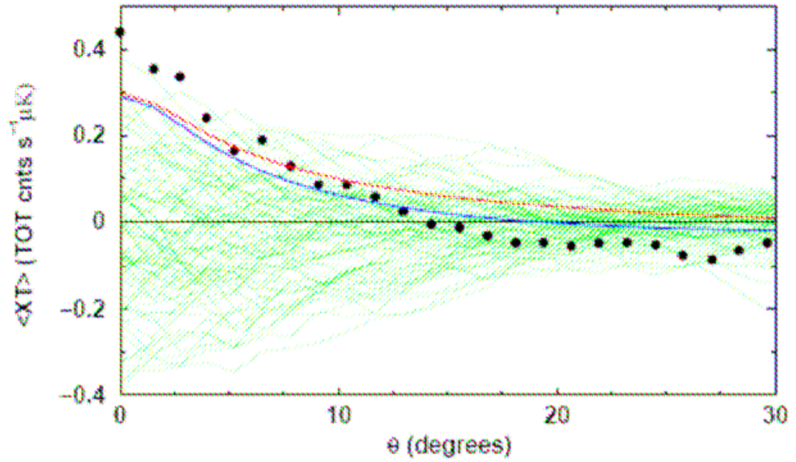
The important step is to get a decent density map of the low-z Universe so as to cross-correlate with the available Microwave Background maps. Once such a map can be made or approximated, the general technique is to smooth both the CMB and large scale structure (LSS) map to a similar resolution and then compute the cross-correlation function

$$CCF(\theta) = \langle (\Delta T_i)(\delta_j) \rangle$$

where θ is the angle between points i and j, ΔT is the temperature fluctuation of the CMB, δ is the over-density, and the average is taken at all pairs of points i and j that are the angle θ apart. So for θ = 0 degrees, the average is taken of all ΔT multiplied by the value of δ at the same point. If the Universe were all matter-dominated, as demonstrated above, large scale structure would not have introduced any net effect on the CMB, and the correlation should average to 0. A Universe with dark energy will have a positive correlation between the temperature of the CMB and directions on the sky with large over-densities of matter at $z<1$.

One of the first important detections of this was by Boughn & Crittenden, in their 2004 paper *A correlation of the cosmic microwave sky with large scale structure*. They used two LSS maps to correlate with the CMB. One was the hard X-ray background (XRB) map made by the HEAO-1 satellite, and the other was the National Radio Astronomy Observatory Very Large Array Sky Survey (NVSS, in a zealous bit of acronymization). After applying masking to remove as much as possible of galactic emission and foreground emission in the WMAP data and correcting for known systematics in the XRB and NVSS maps, they had sky coverages of 68% for WMAP, 56% for NVSS, and 33% for the XRB. While not full-sky, it was still enough data to get decent results in the cross-correlation. They set the maps to be in pixels of 1.3 degrees across.

They used two types of error analysis. First they created Monte Carlo simulations of the CMB sky which would be random and have no ISW signal in them and calculated the cross-correlation of each with the LSS maps. They also took their data maps and rotated them with respect to each other so that the pixels would line up with very different parts of the sky on the other map. The errors generally agreed and gave them a 2.0 – 2.5 σ detection of dark energy using the NVSS map and 2.5 – 3.0 σ using the XRB map. The data from the correlation of the XRB and CMB is presented below.

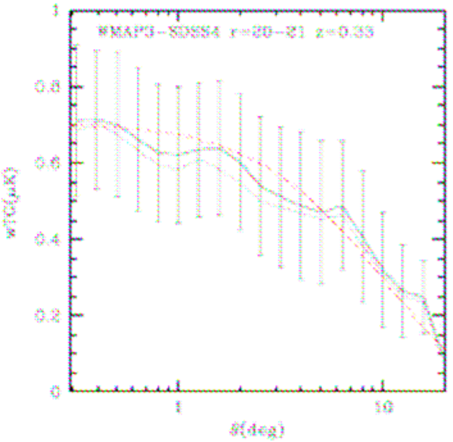


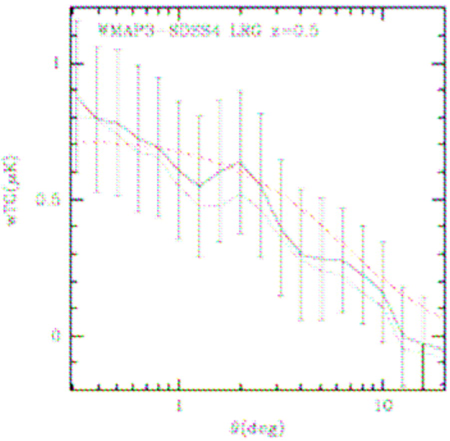
The dots are the data from their analysis, the two solid lines are theoretical predictions of a Ω_Λ=0.72 Universe, one normal and one assuming quadrupole and octupole modes are suppressed, which makes the predicted correlation slightly lower at larger angles. The light lines are the cross-correlations calculated from the Monte Carlo samples. At small angles, the Monte Carlo simulations almost never give correlations as strong as those that Boughn and Crittenden found in their data, implying a likely existence of dark energy.

A recent ISW study with new data was done by Cabré et al, entitled *Cross-Correlation of WMAP* 3^{rd} *year data and the SDSS DR4 galaxy survey: new evidence for Dark Energy*. This group took the latest data from WMAP and correlated it with maps made from one of the biggest large scale structure surveys around, the Sloan Digital Sky Survey. Although their data only covered 13% of the sky, the higher quality of the data ended up giving them a higher significance of detection, getting signal-to-noise of 4.7.

Assuming the number density of galaxies roughly traces out the density of the Universe, they created two LSS maps from the Sloan data. One was a map of galaxies with magnitude of r=20-21 of median redshift z ≈ 0.33, and the other was a map of high-z Luminous Red Galaxies (LRG) of median redshift z ≈ 0.5, which was a sample selected by color. From WMAP they used the V-band (61 GHz) because of the combination of low pixel noise and low emission from the

Galaxy. With this data they were able to smooth their data into maps with pixels only 7’ across. With similar correlation analysis to that described previously, their results were as follows:





Here the solid line shows their data with error bars, the dotted line shows their same analysis with WMAP 1st year data rather than 3rd year, and the dashed line is the prediction for a ΛCDM cosmology with the best fit for Ω_Λ from this data, which was $\Omega_\Lambda = 0.83$. Their signal to noise is ~3.6 for the r=20-21 sample and ~3.0 for the LRG sample, which gives a total S/N of about 4.7. Their error analysis was done multiple ways which they mention, but only explain in another paper not yet released. As well as using multiple simulations and comparing the spread, they also use jack-knife error estimates, where they cut off pieces of data to see how it affects the statistics of the rest, and theoretical estimation. They claim that the errors are all comparable.

They went further in their analysis than Boughn & Crittenden, not satisfied with a mere detection, but also did parameter estimates. By finding the χ^2 for all different values of Ω_Λ, they found a probability distribution for Ω_Λ given their data. Both the r=20-21 sample and the LRG sample were centered on the same value, which is a good sign for consistency. The r=20-21 sample gave slightly narrower constraints, and taken together they give $\Omega_\Lambda=0.83\ ^{+0.02}/_{-0.03}$. They also do 2-dimensional analysis, letting Ω_Λ and w vary, but keeping the Universe flat. They get a wide degeneracy, with lower values of |w| allowing a wider range of Ω_Λ, but overall demanding $0.70 < \Omega_\Lambda < 0.91$ at 1σ and not placing tight constraints on w at all.

The use of the ISW to detect has many advantages. It depends only on simple, well-understood linear physics and is detected on large scales, so does not require high-resolution telescopes. It examines structures at least as big as 100 Mpc across, which is the largest scale to be directly tested. Statistical errors in redshift do not hurt results, and systematic errors tend not to bias the results, so it is a good cross-check of the systematics in other experiments. Since it depends on large scale structure in the dark-energy dominated epoch, only the relatively local density fluctuations will play a part (although theoretically correlating CMB with high-z density maps could show lack of dark energy at earlier times).

However, it also has severe limitations. Because the anisotropies introduced by large scale structure are so much smaller than those imprinted at recombination, the signal is fairly faint and the noise is easily dominated by cosmic variance, especially at the large scales at which the ISW effect happens. As Afshordi deduces, the errors in the cross-correlation signal are at least

$$\Delta C_{gT}^{2}(\ell) \approx \frac{C_{gg}(\ell) C_{TT}(\ell)}{f_{sky}(2\ell+1)}$$

He derives that even with a “perfect” full sky survey of galaxies with known redshifts, the best possible detection will be at 7.5σ. This is not too much higher than has already been reached, and of course perfect full sky surveys are not particularly feasible. Although systematic errors do not generally bias the results, they do dilute them, and to get good signal, Afshordi estimates that the systematic anisotropy of a galaxy survey must be less than 0.1% on the scale of ~10 degrees, while the redshift systematic errors should be less than 0.05. The best detections and parameter estimates of dark energy are almost certainly not going to come from ISW studies in the future, although the ISW effect can still play a complementary role.

The Integrated Sachs-Wolfe effect may have already reached nearly its peak usefulness as a tool to detect the existence of dark energy, especially with the recent analysis done by Cabré et al, but it is certainly a good independent check that the physics world did not go prematurely overboard with excitement about the famous supernovae results. The simplicity of the major equations involved gives it an aesthetic appeal, and though it will likely never fully catch on in course syllabi and powerpoint slides, at least the results are out there.

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